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A NEW CRITERION FOR EARLIEST VISIBILITY OF NEW LUNAR CRESCENT

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Abstract

The single parameter criterion of Yallop (1998) is considered as most authentic guideline for earliest visibility of new lunar crescent for amateur astronomers and the common people searching the new crescent for calendarical purposes. Yallop's criterion is based on the data deduced from Bruin's visibility curves (1977). Bruin developed his curves considering average brightness of the full moon close to the horizon while assuming the crescent as a small disc based on the theory of extinction due to Bamporad (1904). He also used the average brightness of the twilight sky that makes Bruin's visibility curves highly approximate.

In this work the minimum magnitude/brightness of twilight sky has been considered for crescents of various widths that were reported seen, taking into account the elevation above see level the local temperature and relative humidity in line with the algorithm due to Schaefer (1998). Magnitude/brightness of the crescent has been compared with the brightness of the sky to develop the visibility curves and the minimum visibility curves. These curves have lead to a new basic data that consequently lead to a new single parameter semi-empirical criterion for the visibility of new lunar crescent. Analyzing the criterion on the basis of more than 450 evening crescents observations reported by Odeh (2004) new visibility ranges have been suggested and the best time of visibility has been modified.

Keywords: Lunar Crescent, Sky Brightness, Crescent Width.

1. Introduction

The first astrophysical model that studied the earliest visibility of new lunar crescent was due to Bruin (1977), in which the brightness of crescent and that of the twilight sky were taken into consideration. In this model Bruin considered a set of points above horizon where the contrast between crescent brightness and the sky brightness is just in favor of crescent visibility. At the time of sunset (when the depression *s* of the sun below horizon is just zero) the point on the sky, where the contrast between sky brightness and that of the crescent is in favor of crescent visibility, is highest. Points on sky below this point are brighter and the crescent should not be visible below it. As the sun goes down (*s* increases) the altitude *h* of the sky point, where the contrast is just in favor of crescent visibility, decreases while the brightness of the twilight sky keeps decreasing. This *h* is the altitude of a point in sky where the contrast is just in favor of crescent visibility for crescent of particular brightness and not the altitude h_c of the crescent. Plotting *h* against *s* for all such points the Bruin's *visibility curve* (Bruin, 1977) for the crescent is obtained **(Fig. 1)**. In general wider is the crescent brighter it is and must be visible at lower altitudes. Thus for a wider and brighter crescent Bruin's visibility curve would be lower.

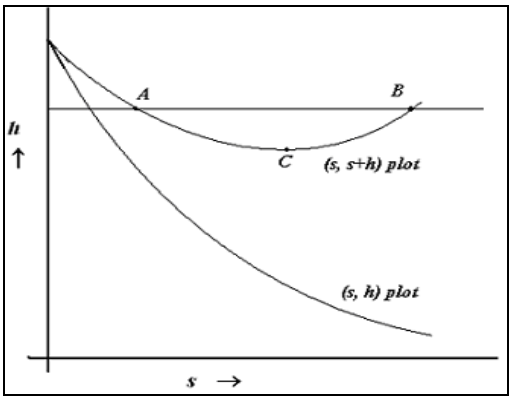

Fig 1: Bruin's Visibility and Minimum Visibility Curve

Apart from the visibility curves or (*s*, *h*) plots Bruin also constructed (*s*, *s* + *h*) plots (Fig 1 of Bruin, 1977) that were called minimum visibility curves by Bruin. For every visibility curve corresponding to a given crescent width there is *minimum visibility curve* which is almost parabolic (Fig. 1). Bruin indicated that during the interval between sunset and the moonset for a new crescent *s* + *h* (equivalent to ARCV, arc of vision that gives the altitude of crescent relative to that of the sun) is almost constant. However, this is true only when *h* represents the altitude h_c of the crescent.

For a particular new crescent the line h = ARCV = $s + h_c$ is a horizontal line on the (s, h) or (s, $s + h$) plot. If ARCV is more than $s + h$ for the minimum or turning point of the corresponding minimum visibility curve (point C on (s, $s + h$) plot) this horizontal line intersects the minimum visibility curve at two points one for smaller value of s (closer to time of sunset) and the other for a larger value of s (closer to time of moonset) shown as points A and B, respectively in Fig 1. Bruin argued that the crescent would become visible when its altitude is that of point A, shall remain visible till its altitude reaches the value of h corresponding to that of point B. Thus Bruin claimed that minimum visibility curve also suggests how long the crescent would remain visible. In Fig 9 of Bruin (1977) both (s, h) and (s, $s + h$) plots are shown for crescents of different widths. Yallop (1998) deduced his basic data from the lowest points (like point C in Fig 1) on the (s, $s + h$) plots in terms of crescent width and ARCV.

Bruin's model considers only the average brightness of the full moon close to horizon and deduces brightness of crescent in the form of a small disc of 3 arc minutes (on the basis of Kooman *et. al.*, 1952). However, the new crescent is only a thin sickle just a few arc seconds at its middle and not a small disc. Moreover, this model considers only average brightness of the twilight sky based on the theory of extinction due to Bemporad (Bemporad, 1904). The brightness of twilight sky actually varies a lot with atmospheric conditions and the seasons. Although the basic idea of developing visibility curves and the minimum visibility curves is very close to reality but the use of actual apparent brightness of crescent and that of twilight sky at each moment according to local conditions would make it more realistic.

Yallop (1998) deduced basic data **(Table 1)** for relative altitude ARCV or $s + h_c$ and crescent width W from the turning points of Bruin's minimum visibility curves interpreting ARCV for the turning point to be the minimum possible ARCV in order that the crescent of width W can be seen.

Table No. 1. Yallop's Basic Data

W	0'·3	0'·5	0'·7	1'	2'	3'
ARCV	10^0·0	8^0·4	7^0·5	6^0·4	4^0·7	4^0·3

This was a major step forward from the other visibility criteria developed in the twentieth century or earlier as these other criteria used basic data in terms of ARCV and DAZ (azimuth of the crescent relative to that of the sun). Finally using least square approximation to fit a cubic polynomial to this basic data (table 1) Yallop obtained the following relation between relative altitude, ARCV and the crescent width W:

$$ACRV = 11.8371 - 6.3226W + 0.7319\,W^2 - 0.1018W^3 \quad (1)$$

Comparing this relation and the actual recorded observations Yallop deduced the parameter q defined below and various visibility conditions on the basis of different ranges of q-values and the observational result

$$q = (ARCV - (11.8371 - 6.3226\,W + 0.7319\,W^2 - 0.1018\,W^3))/10 \quad (2)$$

Thus Yallop's criterion is based on approximate techniques, the approximate minimum visibility curves of Bruin and the ARCV-DAZ based data of Scoch that is restricted to observations that were done mainly from sites within Europe. Yallop concluded from an analysis of the observational data that if $q > 0.216$ the crescent is "Easily Visible", if $-0.014 < q < 0.216$ then crescent is "Visible Under Perfect Weather Conditions" and so on till $q < -0.232$ when crescent is "Not Visible With Telescope". Another remarkable achievement of Yallop was to deduce the best time of new crescent visibility. If T_S is the time of sunset and LAG = difference of time of moonset (T_M) and time of sunset (T_S) then Yallop deduced from the minimum visibility curves of Bruin that the best time T_B of crescent visibility is:

$$T_B = T_S + \frac{4}{9} LAG \quad (3)$$

The coefficient 4/9 of the LAG is deduced from the slope (= 9/4) of the line joining the turning points (vertices or minima) of the minimum visibility curves.

2. The new criterion

In this work we have used the algorithm due to Schaefer (1988) that determines the brightness of sky in terms of "minimum brightness" or "minimum magnitude" that gives the minimum brightness of a celestial object that may be visible on the sky at any time. The algorithm is available on the internet (http://www.go.ednet.ns.ca/~larry/astro/vislimit.html, Bogan, 2004) that we have explored in order to deduce the new visibility curves and the minimum visibility curves. This algorithm uses approximate phase of moon, approximate position of the sun, scattering of light due to air-mass, ozone and contributions from the weather parameters including relative humidity, atmospheric temperature, altitude of site etc. Based on ELP-2000, the lunar theory due to Chapront and Chapront-Touzè (Chapront-Touzé and Chapront, 1991) and a shortened version of VSOP-87 due to Bretagnon and Francou (Bretagnon and Francou, 1988) deduced

by Meeus (Meeus, 1998) we have used the accurate phases of Moon and the position of the Sun in this algorithm. This leads to more accurate values of

the sky brightness and the limiting magnitude (Qureshi, 2007).

We selected a number of actually observed crescents of similar widths at different locations (mostly chosen from the data reported by Schaefer 1988, Odeh, 2004 and Yallop, 1998) and determined the points over twilight sky where the brightness contrast is in favor of crescent visibility. The selected data set is shown in the **(Table 2)**. This table shows the selected observations in order of increasing crescent width (given in last column). The first column gives the observation number (as given by Odeh, 2004) followed by date of observation number in the second column. The rest of the columns show longitude, latitude and altitude of the site and then the estimated temperature and the relative humidity. Some observations recorded at the astronomical observatory, University of Karachi are not assigned any numbers in first column and only 'Karachi' is entered in the column.

Table 2: Circumstances for the day of selected observation for datain table no. 3

Obs No.	Date	Long	Lat	Altitude	Temp	Rel Hum	Width
		Degrees	Degrees	Meters	Celsius	%	Arc sec
120	15/3/1972	-117.6	35.5	930	21	20	14
389	7/1/2000	18.4	-34	200	25	30	16
173	28/1/1979	-81.3	29.9	0	15	25	16
162	9/3/1978	-64.2	44.1	9	3	20	16
433	26/12/2000	18.4	-34	200	20	30	17
598	2/5/2003	-9.5	51.7	50	12	50	27
55	4/6/1875	-2.6	51.5	0	18	75	29.1
269	4/6/1989	-1	50.8	0	14	60	29.1
38	20/2/1871	23.7	38	123	17	60	29.6
108	14/4/1953	5.3	51.1	31	17	75	29.8
89	13/3/1918	5.1	50.2	123	10	80	31
83	31/1/1911	-0.9	51	61	0	45	36
5	20/6/1860	23.7	38	123	27	40	58.9
17	10/1/1864	23.7	38	123	10	65	59.1
51	31/12/1872	23.7	38	123	27	65	59.3
101	28/2/1922	18.5	-33.9	33	23	60	59.8
Karachi	10/4/2005	67	25	0	35	25	60
2	27/10/1859	23.7	38	122	10	55	64.3
87	25/8/1911	2.3	49.9	15	21	65	64.9
16	28/7/1862	23.7	38	122	29	35	67.8
68	31/7/1878	23.7	38	122	29	35	92
Karachi	7/8/2005	67	25	0	38	30	92
23	3/9/1864	23.7	38	122	27	45	93
21	6/6/1864	23.7	38	122	27	40	93
168	5/10/1986	-73.2	40.8	30	18	60	118
63	7/11/1877	23.7	38	122	16	65	118
11	4/11/1861	23.7	38	122	16	65	119
8	8/8/1861	23.7	38	123	29	35	124
Karachi	31/8/2000	67	25	0	35	40	124

For the crescent of same width variations were found in the resulting visibility curves due to site elevations and the weather conditions. For high elevation places the suitable contrast is obtained at lower altitudes points of twilight sky. Similarly the suitable contrast is obtained at lower altitudes for cooler and drier conditions. Thus for crescents of similar width the values of *h* obtained are averaged for a fixed solar depression *s*. The results are presented in table 3.

The top two rows in the table show the crescent widths in arc seconds (first row) and the corresponding apparent magnitude of crescent (second row). The first column in this table shows the solar

depression *s* in degrees below horizon in **(Table 3)**. Columns with heading *h* show the altitude *h* of sky points where the contrast is just in favor of crescent visibility (also in degrees) for the crescent width given at the top of the column at different solar depressions *s*.

Table No. 3: Data for Visibility Curves

Width →	16"	30"	62"	92"	121"
Magnitude →	-4.2	-4.7	-5.3	-5.9	-6.7
s ↓	*h*	*h*	*h*	*h*	*h*
0	18	15.5	13	8	6.5
1	13.5	11.5	9.2	5.5	4.5
2	9.5	8	6.4	3.8	3
3	7.1	5.9	4.2	2.6	2
4	5.6	4.3	2.9	1.8	1.3
5	4.3	3.2	2.1	1.3	1
6	3.55	2.5	1.7	1.1	0.8
7	3.05	2.1	1.5	0.9	0.65
8	2.87	1.9	1.3	0.8	0.55
9	2.7	1.75	1.1	0.7	0.45
10	2.55	1.6	0.95	0.6	0.35
11	2.4	1.45	0.8	0.5	0.3
12	2.3	1.3	0.7	0.4	0.25
13	2.2	1.2	0.6	0.3	0.2
14	2.1	1.1	0.5	0.2	0.15
15	2	1	0.4	0.15	0.1

The resulting visibility curves are shown in the **(Fig. 2)** and the corresponding minimum visibility curves obtained are shown in the **(Fig. 3)**.

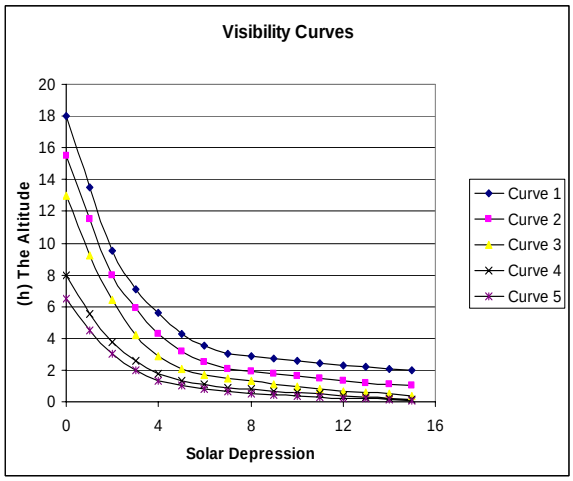

Fig. 2. Visibility Curves

The visibility curves and the minimum visibility curves show almost the same behavior as that obtained by Bruin (Bruin, 1977).

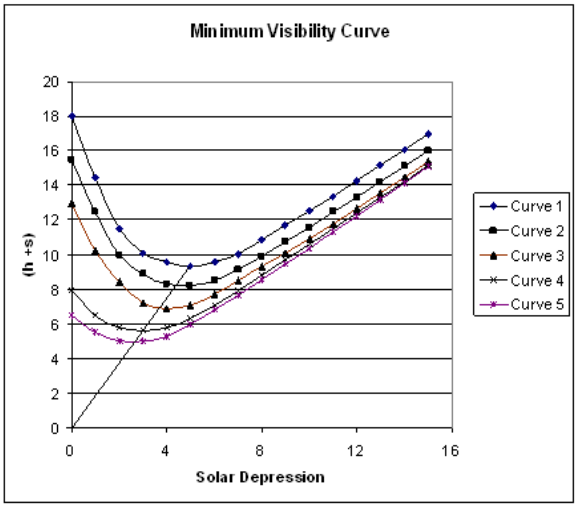

Fig. 3. Minimum Visibility Curves

However, our results are more dependable as more accurate models of sky brightness have been used. From the minima (turning points) of the minimum visibility curves the new basic data has been deduced shown in the following **(Table 4)**. This table shows the width of crescent W in arc seconds and ARCV in degrees. Finally fitting a third degree polynomial using least square approximation ARCV has obtained as following function of crescent width:

Table No. 4. New Basic Data

W	16”	30”	62”	92”	121”
ARCV	$9^0.3$	$8^0.2$	$6^0.9$	$5^0.6$	$4^0.9$

$$ARCV = -0.351964\ W^3 + 2.222075\ W^2 - 5.422643\ W + 10.43418 \quad (4)$$

$$s = (ARCV - 0.351964\ W^3 + 2.222075\ W^2 - 5.422643\ W + 10.43418))/10 \quad (5)$$

On the basis of this relation we define a parameter *s* in order to analyze the observational data to deduce the new visibility criterion as:

The criterion that we have deduced for earliest visibility of new lunar crescent on the basis of our model is that *if s > 0 the crescent may be visible without optical aid otherwise not.* Applying this condition on the observational data reported in literature we obtained *s*-values for a data set of 463 cases in the last column of **(Tab 5)**.

Table No. 5: The s-values for recorded data

Obs.	Date	Lat	Long.	VISIBILITY			ARCL	ARCV	Width	Model	
No.		deg	deg	N	B	T	deg	deg	Arc sec	q-value	s-value
514	12/2/2002	43.9	18.4				5.94	1.56	4.76	-0.98	-0.85
565	4/12/2002	30.9	35.8				4.04	2.58	2.44	-0.9	-0.76
730	10/1/2005	11.2	7.6				5.94	3	5.4	-0.83	-0.7
481	17/9/2001	4.1	73.3				5.23	3.73	4.16	-0.77	-0.63
345	16/2/1999	33.3	44.4				4.47	4.05	2.91	-0.75	-0.61
189	3/1/1984	15.6	35.6				5.51	4.21	4.16	-0.72	-0.59
498	15/11/2001	24.3	54.3				4.62	4.61	3.11	-0.69	-0.55
699	14/10/2004	26.6	50				6.21	4.41	5.62	-0.68	-0.55
247	26/6/1987	-30	-71				8.99	4.02	10.9	-0.67	-0.55
700	14/10/2004	30.4	35.5				6.7	4.37	6.56	-0.68	-0.55
499	15/11/2001	-34	18.4				6.36	4.46	5.87	-0.68	-0.55
500	15/11/2001	31.9	35.8				5.06	4.85	3.73	-0.66	-0.53
566	4/12/2002	10.3	9.8				5.42	4.86	4.38	-0.65	-0.52
626	24/11/2003	32.9	59.2				8.57	4.55	11.2	-0.61	-0.49
664	18/6/2004	-34	18.4				9.79	4.46	12.9	-0.61	-0.49
724	12/12/2004	32.5	3.7				10.07	4.38	15.4	-0.59	-0.48
659	19/5/2004	29.4	48				5.37	5.36	3.88	-0.61	-0.47
567	4/12/2002	6.5	3.4				5.73	5.31	4.89	-0.6	-0.47
627	24/11/2003	32.6	51.7				8.88	4.8	12	-0.58	-0.46
482	17/9/2001	26.6	50				5.62	5.5	4.81	-0.58	-0.45
41	18/6/1871	38	23.7				7.05	5.39	6.7	-0.58	-0.45
106	27/4/1922	-34	18.5				6.13	5.54	5.2	-0.58	-0.44
368	13/7/1999	24.6	46.5				7.79	5.36	9.03	-0.55	-0.43
518	14/3/2002	26.6	50				7.65	5.56	7.86	-0.55	-0.42
519	14/3/2002	26.2	50.5				7.64	5.58	7.83	-0.55	-0.42
444	23/2/2001	-34	18.4				5.98	5.85	4.84	-0.55	-0.42
520	14/3/2002	30.4	35.5				7.99	5.63	8.56	-0.53	-0.41
383	8/12/1999	-34	18.4				9.36	5.58	11.7	-0.51	-0.39
501	15/11/2001	10.3	9.8				6.11	6.11	5.42	-0.52	-0.39
678	16/8/2004	-34	18.4				8.54	5.88	10.1	-0.49	-0.37
515	12/2/2002	-34	18.4				6.49	6.38	5.68	-0.49	-0.36
3	23/1/1860	38	23.7				7.08	6.31	6.75	-0.48	-0.35
53	20/12/1873	38	23.7				11.63	5.39	19.6	-0.45	-0.35
521	14/3/2002	4.1	73.3				7.16	6.52	6.88	-0.46	-0.33
502	15/11/2001	41.1	-74				8.43	6.34	10.3	-0.44	-0.32
522	14/3/2002	32.5	3.7				8.76	6.38	10.3	-0.44	-0.32
660	19/5/2004	32.5	3.7				6.74	6.72	6.12	-0.45	-0.32
510	15/12/2001	32.6	51.7				8.47	6.49	10.1	-0.43	-0.31
612	26/9/2003	26.6	50				7.42	6.85	8.22	-0.41	-0.29
7	7/8/1861	38	23.7				16.03	4.97	37.5	-0.32	-0.29
381	8/11/1999	32	35.9				7.03	7	6.68	-0.41	-0.29
382	8/11/1999	31.8	34.7				7.06	7.03	6.74	-0.41	-0.28
424	26/11/2000	32.6	51.7				7.51	7.01	7.76	-0.4	-0.28
578	3/1/2003	32	51.9				9.96	6.53	14.3	-0.38	-0.27

613	26/9/2003	30.4	35.5				7.89	6.95	9.31	-0.39	-0.27
Obs.	Date	Lat	Long	VISIBILITY			ARCL	ARCV	Width	Model	
No.		deg	deg	N	B	T	deg	deg	Arc sec	q-value	s-value
220	31/12/1986	39	-77				12.38	6.03	23.3	-0.35	-0.26
305	2/3/1995	-34	18.4				15.46	5.45	33.5	-0.31	-0.26
720	13/11/2004	36.1	50.3				13.57	5.91	27.6	-0.32	-0.25
507	15/12/2001	31.8	35.2				9.05	7.02	11.5	-0.36	-0.25
702	14/10/2004	6.5	3.4				8.03	7.19	9.42	-0.37	-0.24
335	28/3/1998	-34	18.4				8.74	7.03	11.7	-0.36	-0.24
614	26/9/2003	-34	18.4				8.53	7.1	10.9	-0.36	-0.24
371	13/7/1999	43.3	-79.9				12.71	6.3	23.9	-0.31	-0.23
425	26/11/2000	32	35.9				7.98	7.39	8.73	-0.35	-0.23
192	2/2/1984	15.6	35.6				8.64	7.4	10	-0.34	-0.22
703	14/10/2004	-34	18.4				7.77	7.5	8.81	-0.34	-0.22
646	21/3/2004	-34	18.4				9.55	7.21	12.7	-0.33	-0.22
523	14/3/2002	27.7	-11.3				9.15	7.33	11.2	-0.34	-0.22
679	16/8/2004	35.7	51.3				8.21	7.49	9.3	-0.34	-0.22
257	16/4/1988	37.2	-84.1				7.7	7.61	8.73	-0.33	-0.21
369	13/7/1999	-34	18.4				7.92	7.58	9.34	-0.33	-0.21
370	13/7/1999	-34	18.4				7.92	7.58	9.34	-0.33	-0.21
400	6/3/2000	43.3	-79.9				10.09	7.21	14.4	-0.32	-0.2
680	16/8/2004	30.2	57.1				8	7.64	8.84	-0.33	-0.2
384	8/12/1999	-4	39.7				8.39	7.69	9.44	-0.32	-0.19
389	7/1/2000	-34	18.4	V			10.96	7.19	16.2	-0.3	-0.19
388	7/1/2000	23.7	90.4				7.99	7.84	8.62	-0.31	-0.19
476	19/8/2001	38.6	48.2				8.89	7.61	12.1	-0.3	-0.18
211	23/11/1984	15.6	35.6				9.21	7.6	12.5	-0.3	-0.18
390	7/1/2000	-32	20.8				10.86	7.36	15.9	-0.29	-0.18
422	27/10/2000	38.8	-77.2				8.69	7.78	10.6	-0.3	-0.18
681	16/8/2004	26.6	50				8.15	7.9	9.18	-0.3	-0.18
572	3/1/2003	26.6	49.8				10.15	7.47	14.8	-0.29	-0.18
583	3/3/2003	-34	18.4				8.58	7.88	10.1	-0.29	-0.17
216	20/4/1985	37.2	-84.1				8.68	7.92	10.1	-0.29	-0.17
706	14/10/2004	25.8	-80.2				10.95	7.37	17.5	-0.27	-0.17
647	21/3/2004	29.4	48				8.68	7.89	10.5	-0.29	-0.17
408	4/5/2000	-34	18.4				8.28	7.92	10.2	-0.29	-0.17
20	6/5/1864	39.6	26.2				9.08	7.8	11.8	-0.28	-0.16
538	11/6/2002	31.9	35.8				8.78	7.87	10.9	-0.28	-0.16
426	26/11/2000	-4	39.7				8.3	7.99	9.44	-0.29	-0.16
682	16/8/2004	32	35.9			V	8.58	7.96	10.2	-0.28	-0.16
648	21/3/2004	26.6	50				8.61	7.98	10.4	-0.28	-0.16
477	19/8/2001	29.5	56.8		V		8.47	7.95	10.9	-0.28	-0.16
480	19/8/2001	32.5	51.3				8.7	7.94	11.5	-0.27	-0.15
683	16/8/2004	30.2	35.5				8.57	8.05	10.2	-0.27	-0.15
549	7/9/2002	31.1	56.5		V		8.44	8	10.8	-0.27	-0.15
386	8/12/1999	36.8	10.4		V		8.68	8.07	10.1	-0.27	-0.15
385	8/12/1999	26.2	32.7				8.24	8.15	9.1	-0.28	-0.15
427	26/11/2000	32.5	3.7				8.91	8.05	10.9	-0.27	-0.15

299	3/12/1994	-34	18.4				11.14	7.48	18.8	-0.24	-0.15
Obs.	Date	Lat	Long	VISIBILITY			ARCL	ARCV	Width	Model	
No.		deg	deg	N	B	T	deg	deg	Arc sec	q-value	s-value
551	7/9/2002	32.5	51.3				8.62	8.05	11.3	-0.26	-0.14
557	5/11/2002	29.9	56.2		V		10.11	7.82	15.4	-0.24	-0.14
638	22/1/2004	30	51.7		V		10.88	7.7	17.5	-0.24	-0.13
649	21/3/2004	38	23.7				9.44	8.07	12.4	-0.25	-0.13
428	26/11/2000	34	-6.8				9.19	8.14	11.6	-0.25	-0.13
318	7/5/1997	36	50.8				11.16	7.68	18	-0.23	-0.13
10	5/10/1861	38	23.7				20.2	5.54	61.5	-0.05	-0.13
207	25/9/1984	15.6	35.6				8.42	8.24	10.8	-0.25	-0.13
558	5/11/2002	30.1	52.1		V		10.26	7.9	15.9	-0.23	-0.12
650	21/3/2004	30.4	35.5				9.06	8.24	11.5	-0.24	-0.12
308	22/12/1995	36.1	50.7				8.33	8.32	10.6	-0.24	-0.12
559	5/11/2002	29.6	52.5		V		10.25	7.95	15.9	-0.23	-0.12
455	25/3/2001	-34	18.4	V			9.06	8.3	11.3	-0.24	-0.12
629	24/11/2003	41.5	-112		V		15.19	6.77	35	-0.16	-0.12
328	30/12/1997	-34	18.4		V		14.04	7.15	28.6	-0.18	-0.12
429	26/12/2000	29.6	52.5				9.36	8.29	11.8	-0.23	-0.12
528	13/4/2002	32.6	51.7				9.89	8.18	13.2	-0.23	-0.12
391	7/1/2000	32.7	52.3		V		9.01	8.36	11	-0.24	-0.12
340	19/12/1998	31.9	35.8				8.55	8.47	10	-0.23	-0.11
374	10/9/1999	31.8	34.7				9.87	8.2	13.7	-0.22	-0.11
725	12/12/2004	11.2	7.6				10.37	8.02	16.4	-0.22	-0.11
589	2/4/2003	32.6	51.6				9.6	8.32	12.4	-0.22	-0.11
726	12/12/2004	10.3	9.8				10.31	8.06	16.2	-0.21	-0.11
274	25/2/1990	35.6	-83.5	V		V	8.53	8.51	10.7	-0.22	-0.1
275	25/2/1990	35.6	-83.5			V	8.53	8.51	10.7	-0.22	-0.1
276	25/2/1990	35.6	-83.5				8.53	8.51	10.7	-0.22	-0.1
101	31/10/1921	-34	18.5				9.84	8.27	14	-0.21	-0.1
326	2/10/1997	31.8	34.7				10.71	8.17	15.5	-0.21	-0.1
375	10/9/1999	30.4	35.5			V	9.84	8.31	13.6	-0.21	-0.1
430	26/12/2000	32.6	35.9				9.81	8.39	13	-0.21	-0.1
560	5/11/2002	31.9	35.8		V		10.86	8.04	17.8	-0.2	-0.1
321	7/5/1997	32.7	52.3		V		11.06	8.06	17.7	-0.2	-0.1
714	13/11/2004	32	35.9			V	14.21	7.27	30.2	-0.16	-0.1
573	3/1/2003	32.5	3.7		V		11.61	7.96	19.4	-0.19	-0.09
524	13/4/2002	29.6	52.5				9.85	8.42	13.1	-0.21	-0.09
478	19/8/2001	30.2	35.5		V	V	9.24	8.47	13	-0.2	-0.09
281	24/5/1990	35.6	-83.5			V	8.67	8.66	11.4	-0.2	-0.08
431	26/12/2000	26.6	50				9.48	8.64	12.1	-0.2	-0.08
341	18/1/1999	-34	18.4	V			13.31	7.79	25	-0.15	-0.07
15	29/4/1862	38	23.7				8.86	8.85	10.5	-0.19	-0.07
39	20/4/1871	38	23.7				11.09	8.4	16.6	-0.17	-0.07
105	29/3/1922	-34	18.5				12.91	8	22.6	-0.16	-0.07
432	26/12/2000	30.2	35.5		V	V	9.87	8.67	13.1	-0.18	-0.07
312	20/1/1996	34.1	-118			V	8.92	8.78	12.1	-0.18	-0.06
184	5/11/1983	15.6	35.6				9.29	8.78	12.4	-0.18	-0.06

264	5/5/1989	42.7	-84.8			V	8.91	8.85	11.9	-0.18	-0.06
Obs.	Date	Lat	Long	VISIBILITY			ARCL	ARCV	Width	Model	
No.		deg	deg	N	B	T	deg	deg	Arc sec	q-value	s-value
265	5/5/1989	42.7	-84.8				8.91	8.85	11.9	-0.18	-0.06
688	15/9/2004	36.6	59		V		12.44	8.14	22	-0.15	-0.06
212	23/11/1984	34	-81			V	13.24	7.92	25.8	-0.13	-0.06
433	26/12/2000	-34	18.4	V			11.34	8.49	17.3	-0.16	-0.06
592	2/5/2003	-34	18.4				12.69	8.2	21.6	-0.15	-0.06
266	5/5/1989	43	-85.7			V	8.95	8.9	12	-0.17	-0.05
199	1/5/1984	37.2	-84.1				9.99	8.77	13.8	-0.17	-0.05
137	1/7/1973	-44	171		V		10.61	8.55	17	-0.16	-0.05
338	21/9/1998	31.8	35.2				10.97	8.61	16.2	-0.16	-0.05
615	26/9/2003	41.8	-112		V		13.01	8.02	25.3	-0.13	-0.05
434	26/12/2000	-32	20.8	V			11.23	8.62	17	-0.15	-0.05
435	26/12/2000	-32	20.8		V		11.23	8.64	17	-0.15	-0.04
260	14/6/1988	37.2	-84.1				9.15	9.09	11.6	-0.16	-0.04
94	8/2/1921	36.5	-6.2				9.25	9.1	11.6	-0.16	-0.04
301	1/1/1995	33	-106			V	9.05	9.05	12.3	-0.15	-0.04
387	8/12/1999	6.5	3.4				9.27	9.14	11.5	-0.15	-0.03
586	2/4/2003	30.2	35.5		V	V	10.05	9	13.6	-0.14	-0.03
286	20/9/1990	31.8	34.7	V			19.64	6.85	52.7	0.007	-0.03
324	4/8/1997	31.3	35.2				15.1	7.92	30.6	-0.09	-0.03
689	15/9/2004	35.7	51.4				12.69	8.4	22.9	-0.11	-0.03
95	8/2/1921	38.8	-9.1				9.31	9.22	11.7	-0.14	-0.02
316	8/2/1997	-34	18.4	V			16.1	7.51	39.3	-0.05	-0.02
600	31/5/2003	26	-80.3				9.42	9.25	12	-0.14	-0.02
319	7/5/1997	31.8	34.9	V			11.64	8.74	19.6	-0.11	-0.01
633	24/12/2003	49.6	8.7	V	V		18.11	7.21	48.9	0.01	-0.01
690	15/9/2004	34.7	50.9				12.7	8.58	22.9	-0.09	-0.01
727	12/12/2004	32.4	-111	V	V	V	14.43	8.13	31.6	-0.06	-0
51	27/4/1873	38	23.7				10.21	9.23	15.1	-0.11	0.003
364	14/6/1999	29.6	52.5				12.62	8.67	24	-0.07	0.007
134	15/3/1972	35.5	-118				9.69	9.35	14.2	-0.1	0.008
135	15/3/1972	35.5	-118		V		9.69	9.35	14.2	-0.1	0.008
2	27/10/1859	38	23.7	V			21.43	6.8	65	0.109	0.008
508	15/12/2001	-4	39.7				9.48	9.48	12.6	-0.11	0.009
392	7/1/2000	-4	39.7				9.89	9.44	13.2	-0.1	0.01
272	1/10/1989	31.3	34.6	V			19.52	7.34	50.7	0.038	0.011
416	31/7/2000	6.5	3.4	V			9.58	9.41	13.9	-0.1	0.011
691	15/9/2004	33.3	50.1		V		12.73	8.82	23	-0.07	0.016
314	21/1/1996	-34	18.4	V			17.89	7.51	48.1	0.032	0.018
337	26/5/1998	31.8	35.2			V	13.12	8.72	25.5	-0.06	0.021
412	2/7/2000	2.3	102		V	V	9.69	9.5	14.3	-0.09	0.024
248	26/6/1987	42.7	-84.5		V		10.47	9.49	14.7	-0.08	0.026
415	2/7/2000	32.6	51.7		V		12.3	8.92	23	-0.06	0.026
48	3/10/1872	38	23.7				12.75	8.97	22.8	-0.06	0.029
365	14/6/1999	29.4	48				12.79	8.86	24.7	-0.05	0.03
672	18/6/2004	33.3	50				9.93	9.68	13.2	-0.08	0.034

639	22/1/2004	32.5	3.7	V	V		12.49	9.03	23	-0.05	0.037
Obs.	Date	Lat	Long	VISIBILITY			ARCL	ARCV	Width	Model	
No.		deg	deg	N	B	T	deg	deg	Arc sec	q-value	s-value
304	31/1/1995	35.6	51.3		V		9.82	9.66	14	-0.07	0.038
185	5/11/1983	37.2	-84.1			V	13.11	8.95	24.6	-0.04	0.039
665	18/6/2004	26.6	50				9.81	9.77	12.9	-0.07	0.04
45	6/7/1872	39	23.7				11.38	9.46	17.3	-0.06	0.042
655	20/4/2004	5	115				10.03	9.73	13.7	-0.07	0.042
666	18/6/2004	28.4	48				9.9	9.82	13.1	-0.07	0.047
334	27/2/1998	-34	18.4	V			14.23	8.69	30.6	-0.01	0.049
667	18/6/2004	24.6	46.5				9.87	9.86	13.1	-0.06	0.051
249	26/6/1987	37.2	-84.1			V	10.33	9.83	14.3	-0.05	0.057
36	12/5/1869	38	23.7				13.48	9.1	25.4	-0.02	0.059
315	13/10/1996	31.8	34.9	V			12.72	9.27	22.8	-0.03	0.059
282	24/5/1990	31.6	-111			V	9.86	9.86	14.8	-0.05	0.063
283	24/5/1990	32.4	-111				9.88	9.87	14.8	-0.04	0.065
393	7/1/2000	34	-6.8				10.8	9.81	15.8	-0.04	0.065
484	17/10/2001	2.3	102		V	V	9.93	9.92	14.6	-0.04	0.068
267	5/5/1989	39.7	-106			V	9.93	9.92	14.8	-0.04	0.069
485	17/10/2001	32.6	51.7		V		11.47	9.66	19.5	-0.02	0.077
593	2/5/2003	5	115			V	10.16	10.1	13.9	-0.03	0.078
668	18/6/2004	32	35.9			V	10.31	10.1	14.2	-0.03	0.079
436	26/12/2000	-4	39.7				10.23	10.1	14.1	-0.03	0.082
290	15/2/1991	33.4	73.1	V			10.17	10.1	14.5	-0.02	0.087
443	25/1/2001	32.6	51.7				11.66	9.86	18.2	-0.01	0.088
250	26/6/1987	39.8	-105				10.99	10.1	16.2	-0.01	0.094
692	15/9/2004	28.4	48				12.79	9.6	23.3	0.011	0.096
376	10/9/1999	38.8	-77		V		13.54	9.45	25.6	0.019	0.096
621	26/10/2003	32.9	59.2		V		14.55	9.08	32.1	0.043	0.096
603	30/6/2003	33.7	58.4				10.9	10.1	16.4	-0.01	0.097
285	24/5/1990	34.2	-118	V		V	10.17	10.1	15.7	-0.01	0.098
284	24/5/1990	34.2	-118			V	10.17	10.1	15.7	-0.01	0.098
511	14/1/2002	29.6	52.5				12.11	9.87	20	0.006	0.101
715	13/11/2004	4.9	115		V		11.62	9.86	20.2	0.008	0.102
251	26/6/1987	40.7	-112		V		11.19	10.1	16.8	0	0.105
88	28/11/1913	-34	18.5	V			10.25	10.3	15.3	-0	0.106
162	9/3/1978	45.1	-64.2	V			10.73	10.2	16.6	0.002	0.107
486	17/10/2001	29.6	52.5		V		11.47	9.97	19.5	0.011	0.107
256	19/1/1988	32.2	-111			V	12.22	9.76	22.7	0.022	0.108
252	26/6/1987	30	-100				10.45	10.4	14.7	0.003	0.113
194	3/3/1984	15.6	35.6				10.9	10.3	15.9	0.008	0.114
693	15/9/2004	26.6	50				12.72	9.82	23	0.031	0.116
622	26/10/2003	33.3	50.1	V	V		14.9	9.19	33.6	0.068	0.116
594	2/5/2003	3.2	102			V	10.55	10.4	14.9	0.009	0.118
616	26/9/2003	32.4	-111	V	V	V	13.14	9.68	25.8	0.043	0.119
630	24/11/2003	6.5	3.4				11.31	10.1	19.5	0.023	0.119
694	15/9/2004	26.2	50.5		V		12.71	9.87	23	0.035	0.12
503	16/11/2001	49.6	8.7	V	V		17.76	8.7	45	0.123	0.123

437	25/1/2001	29.6	52.5		V		11.68	10.2	18.3	0.024	0.123
Obs.	Date	Lat	Long	VISIBILITY			ARCL	ARCV	Width	Model	
No.		deg	deg	N	B	T	deg	deg	Arc sec	q-value	s-value
413	2/7/2000	30.4	35.5			V	12.89	9.75	25.3	0.045	0.123
252	26/6/1987	30	-100				10.63	10.4	15.1	0.016	0.124
553	7/10/2002	49.6	8.7	V	V		18.07	8.53	49.4	0.145	0.125
320	7/5/1997	-34	18.4	V			11.46	10.2	19	0.027	0.125
623	26/10/2003	32.6	51.7		V		14.85	9.31	33.4	0.078	0.127
174	28/1/1979	42	-91.7		V		10.63	10.3	17.2	0.025	0.128
332	28/1/1998	29.8	-95.4		V		10.44	10.4	16.2	0.023	0.129
525	13/4/2002	30.5	-9.7		V		11.65	10.3	18.4	0.029	0.129
404	5/4/2000	5.3	103		V		10.62	10.4	16.4	0.024	0.129
487	17/10/2001	31.9	35.8			V	12.03	10.1	21.4	0.041	0.131
173	28/1/1979	29.9	-81.3	V			10.42	10.4	16.6	0.028	0.132
543	9/8/2002	2.3	102			V	10.49	10.4	16.4	0.029	0.133
176	28/1/1979	42	-93.6	V			10.7	10.4	17.5	0.034	0.135
475	21/7/2001	32.6	51.7				12.27	10	22.8	0.05	0.137
175	28/1/1979	29.7	-82.4		V		10.47	10.5	16.7	0.034	0.138
177	28/1/1979	38.7	-90.3		V		10.64	10.5	17.2	0.039	0.142
695	15/9/2004	32.5	3.7		V		14.29	9.72	29	0.078	0.143
28	24/6/1865	38	23.7				18.57	8.75	48.6	0.161	0.144
488	17/10/2001	26.6	50				11.57	10.3	19.8	0.051	0.146
253	26/6/1987	33.5	-112	V			11.01	10.6	16.2	0.043	0.148
218	12/12/1985	-32	20.8	V			10.78	10.5	17.5	0.048	0.15
405	5/4/2000	32.6	51.7		V		12.43	10.2	22.5	0.064	0.151
716	13/11/2004	36.8	-81.8	V	V		18.66	8.7	52	0.185	0.153
16	28/7/1862	38	23.7				22.29	8.15	68.6	0.273	0.154
417	28/9/2000	-34	18.4	V			12.48	10.3	22.5	0.069	0.156
418	28/9/2000	-34	18.4	V	V		12.48	10.3	22.5	0.069	0.156
396	6/2/2000	-34	18.4	V	V		14.06	9.99	27.3	0.088	0.159
102	30/12/1921	-34	18.5				17.96	9.13	43.6	0.154	0.16
163	9/3/1978	42.7	-73.8	V	V		11.08	10.6	17.7	0.059	0.16
103	29/1/1922	-34	18.5				19.55	8.85	50.9	0.191	0.163
347	18/3/1999	36	50.8	V			12.12	10.4	21.9	0.076	0.165
394	7/1/2000	6.5	3.4				10.87	10.8	16	0.062	0.169
348	18/3/1999	-34	18.4	V	V		13.35	10.2	26.6	0.099	0.172
349	18/3/1999	-34	18.4	V			13.35	10.2	26.6	0.101	0.174
33	5/2/1867	38	23.7				10.96	10.8	16.7	0.071	0.175
142	18/2/1977	43.8	-87.7	V	V		10.88	10.9	16.6	0.072	0.176
489	17/10/2001	24.6	46.5			V	11.71	10.6	20.3	0.084	0.177
669	18/6/2004	32.5	3.7		V		11.25	10.9	17	0.075	0.178
306	28/6/1995	-30	-71	V			10.94	10.9	16.1	0.075	0.181
544	9/8/2002	-34	18.4	V			13.09	10.3	25.6	0.104	0.181
607	28/8/2003	5.3	103		V	V	10.89	10.9	17.3	0.081	0.183
85	31/1/1911	51	-0.9	V			16.4	9.68	37.7	0.155	0.186
96	8/2/1921	42.3	-71.1	V			10.98	11	16.3	0.081	0.186
22	4/8/1864	38	23.7				23.81	8.34	75.1	0.346	0.19
471	21/7/2001	4.1	73.3	V	V		10.93	10.9	18.1	0.093	0.193

608	28/8/2003	32.6	51.7		V		12.78	10.6	23.8	0.112	0.194
Obs.	Date	Lat	Long	VISIBILITY			ARCL	ARCV	Width	Model	
No.		deg	deg	N	B	T	deg	deg	Arc sec	q-value	s-value
472	21/7/2001	32.6	35.9				12.89	10.5	25.2	0.117	0.195
31	17/1/1866	38	23.7				11.01	11	17.3	0.093	0.196
512	14/1/2002	24.6	46.5		V		12.38	10.8	20.9	0.103	0.196
89	16/3/1915	49.4	8.7	V			11.16	11	17.1	0.093	0.197
438	25/1/2001	24.6	46.5		V	V	11.93	10.9	19	0.104	0.201
473	21/7/2001	31.9	35.8	V		V	12.88	10.6	25.1	0.126	0.203
419	28/9/2000	26.2	32.7	V			11.93	10.9	20.5	0.111	0.204
624	26/10/2003	29.4	48		V		15.04	10.1	34.2	0.163	0.208
574	3/1/2003	-34	18.4	V			12.26	10.8	21.6	0.119	0.209
456	24/4/2001	32.6	51.7	V	V		12.89	10.8	23.6	0.129	0.212
323	5/7/1997	-34	18.5	V			11.24	11.2	17.3	0.111	0.214
617	27/9/2003	49.6	8.7	V	V		22.05	8.65	72.2	0.354	0.214
79	7/12/1885	50.6	5.7	V			13.51	10.7	24.7	0.135	0.215
93	19/4/1920	43.5	7	V			11.94	10.9	21.3	0.125	0.216
350	18/3/1999	29.6	52.5	V	V		12.06	11	21.7	0.131	0.22
420	28/9/2000	32.5	3.7	V	V		12.87	10.9	23.9	0.142	0.224
545	9/8/2002	32.6	51.7	V	V	V	12.67	10.9	24	0.144	0.226
414	2/7/2000	-32	20.8	V	V		12.33	10.9	23.1	0.142	0.227
406	5/4/2000	32	35.9			V	12.97	10.8	24.5	0.147	0.227
397	6/2/2000	32.6	51.7	V	V		12.33	11.1	21	0.143	0.235
167	9/3/1978	40.5	-89	V			11.63	11.2	19.5	0.138	0.235
168	9/3/1978	40.5	-89		V		11.63	11.2	19.5	0.138	0.235
609	28/8/2003	32.4	36.2			V	13.3	10.9	25.8	0.164	0.24
297	23/2/1993	-34	18.4	V			24.33	8.81	78.4	0.421	0.245
398	6/2/2000	36.2	37.2	V	V		12.76	11.2	22.5	0.16	0.247
342	18/1/1999	28.8	43.7	V	V		11.64	11.4	19.2	0.152	0.249
83	1/5/1908	44.1	3.1	V			14.76	10.7	30.8	0.194	0.252
561	5/11/2002	-34	18.4	V			12.4	11.2	23.2	0.168	0.253
122	8/12/1942	40.7	-74	V	V		12.56	11.2	24	0.173	0.255
595	2/5/2003	38.2	46	V			12.63	11.3	21.4	0.165	0.255
372	12/8/1999	32.7	52.3				15.37	10.6	33.9	0.209	0.255
288	18/12/1990	33.4	73.1	V			14.57	10.9	28.5	0.189	0.256
407	5/4/2000	-34	18.4	V			13.29	11.1	25.7	0.18	0.256
87	25/8/1911	49.9	2.3	V			21.5	9.26	66.2	0.364	0.258
490	17/10/2001	-34	18.4	V			13.09	11.1	25.3	0.181	0.259
223	28/4/1987	26.7	-81.1	V	V		11.6	11.5	18.6	0.16	0.259
222	28/4/1987	38.9	-77	V	V		11.64	11.5	18.8	0.161	0.26
227	28/4/1987	38.9	-77.1	V	V		11.64	11.5	18.8	0.161	0.26
224	28/4/1987	38.9	-77.1				11.64	11.5	18.8	0.161	0.26
439	25/1/2001	32.5	3.7				13.12	11.3	23	0.174	0.26
421	28/9/2000	43.3	-79.9	V	V		15.64	10.6	35.1	0.22	0.262
343	18/1/1999	26.1	44		V		11.67	11.5	19.3	0.166	0.263
529	13/5/2002	5	115		V	V	11.6	11.6	18.6	0.166	0.265
229	28/4/1987	28	-82.5				11.65	11.6	18.8	0.17	0.268
229	28/4/1987	28	-82.5				11.65	11.6	18.8	0.17	0.268

108	27/5/1922	-34	18.5	V			12.3	11.5	21.5	0.18	0.269
Obs.	Date	Lat	Long	VISIBILITY			ARCL	ARCV	Width	Model	
No.		deg	deg	N	B	T	deg	deg	Arc sec	q-value	s-value
463	22/6/2001	49.6	8.7	V	V		18.31	10	49.9	0.299	0.275
457	24/4/2001	32.6	35.9				13.41	11.3	25.5	0.2	0.277
598	2/5/2003	51.7	-9.5	V	V	V	14.61	11.1	28.6	0.21	0.277
351	18/3/1999	31.9	35.8	V	V		12.69	11.4	24	0.196	0.277
596	2/5/2003	32.6	51.7	V	V		12.37	11.6	20.5	0.184	0.277
640	22/1/2004	-34	18.4	V			12.96	11.3	24.7	0.199	0.278
602	30/6/2003	32.5	3.7		V	V	12.6	11.5	22	0.192	0.28
119	13/6/1934	55.6	33.9	V	V		19.04	10.1	48.5	0.298	0.281
352	18/3/1999	31.8	35.2	V			12.71	11.4	24.1	0.2	0.282
40	20/5/1871	38	23.7				14.2	11.2	27	0.211	0.283
656	20/4/2004	27.3	62.4		V		11.87	11.8	19.2	0.186	0.283
76	30/3/1881	51.5	-2.6	V			11.81	11.7	20	0.189	0.284
230	28/4/1987	36.2	-81.7	V	V		11.8	11.8	19.3	0.188	0.284
458	24/4/2001	31.9	35.8	V		V	13.4	11.4	25.5	0.208	0.285
353	18/3/1999	24.6	46.5		V	V	12.28	11.6	22.5	0.202	0.289
445	24/2/2001	51.7	7.2	V	V		15.98	10.9	34.5	0.246	0.291
597	2/5/2003	27.7	54.4	V	V		12.23	11.8	20	0.2	0.294
634	24/12/2003	33.4	73.1	V	V		16.21	10.7	39.3	0.27	0.295
651	21/3/2004	36.8	-81.8	V			12.78	11.6	22.7	0.209	0.296
81	19/4/1901	50.7	-2.8	V			13.14	11.5	26.2	0.227	0.301
145	9/1/1978	38.9	-76.9		V		11.9	11.8	21.4	0.211	0.302
547	9/8/2002	30.4	35.5	V		V	13.23	11.5	26.1	0.229	0.303
732	11/1/2005	43.9	18.4	V			17.35	10.5	45.6	0.308	0.305
138	21/12/1976	42.7	-83.6	V			12.6	11.7	23.6	0.225	0.308
657	20/4/2004	32.8	51		V		12.29	11.9	20.5	0.217	0.31
58	22/6/1876	38	23.7				12.88	11.7	25.2	0.238	0.316
635	24/12/2003	35.7	51.3	V	V	V	16.98	10.8	43.1	0.313	0.321
146	9/1/1978	41.9	-87.6	V			12.23	12	22.6	0.24	0.326
150	9/1/1978	43	-89.8	V	V		12.28	11.9	22.8	0.24	0.327
708	15/10/2004	32.9	59.2	V	V	V	19.01	10.4	52.8	0.365	0.328
610	28/8/2003	32.5	3.7		V		14.42	11.5	30.3	0.269	0.329
147	9/1/1978	36	-79.8	V	V		12.07	12	22	0.241	0.329
148	9/1/1978	36	-79.8	V			12.07	12	22	0.241	0.329
373	12/8/1999	31.8	34.7	V			15.98	11.2	36.6	0.293	0.329
570	5/12/2002	35.7	51.4	V	V		16.94	10.9	42.1	0.317	0.33
269	4/6/1989	50.8	-1	V			14.49	11.5	30.5	0.27	0.33
55	4/6/1875	51.5	-2.6	V			14.24	11.5	30.6	0.27	0.33
195	3/3/1984	37.2	-84.1			V	14.17	11.7	26.9	0.258	0.331
188	5/12/1983	37.2	-84.1			V	16.98	11	39.8	0.311	0.333
491	17/10/2001	-4	39.7				12.12	12.1	21.8	0.247	0.336
144	11/12/1977	47.8	20	V			13.93	11.6	29.5	0.274	0.338
201	31/5/1984	15.6	35.6				12.19	12.2	21.2	0.247	0.338
670	18/6/2004	47.6	-118	V	V		15.27	11.6	31.2	0.285	0.341
151	9/1/1978	34	-81.1	V	V		12.15	12.1	22.4	0.255	0.343
139	21/12/1976	42	-91.6	V	V		12.91	12	24.8	0.268	0.347

339	21/10/1998	31.8	34.7	V			14.08	11.9	26.5	0.276	0.35
Obs.	Date	Lat	Long	VISIBILITY			ARCL	ARCV	Width	Model	
No.		deg	deg	N	B	T	deg	deg	Arc sec	q-value	s-value
38	20/2/1871	38	23.7	V			14.49	11.8	29.6	0.289	0.351
378	10/10/1999	32	35.9	V		V	14.27	11.9	27.8	0.284	0.354
153	9/1/1978	29.9	-81.3	V			12.23	12.2	22.6	0.268	0.355
155	9/1/1978	41.6	-93.6	V			12.46	12.2	23.5	0.271	0.355
377	10/10/1999	5.3	103	V	V	V	12.37	12.4	20.9	0.263	0.355
187	5/12/1983	15.6	35.6				13.41	12.1	25	0.277	0.356
156	9/1/1978	33.9	-84.3	V			12.27	12.3	22.8	0.271	0.357
710	15/10/2004	32.6	51.7	V	V		19.29	10.7	54.3	0.402	0.358
709	15/10/2004	32.6	51.6	V	V		19.3	10.7	54.3	0.402	0.358
550	7/9/2002	10.7	-61.5	V			12.44	12.2	23.5	0.274	0.358
575	3/1/2003	33.9	-118		V		15.81	11.5	35.6	0.319	0.359
157	9/1/1978	27.7	-82.7	V	V		12.32	12.3	23	0.278	0.364
80	29/5/1900	38.7	-0.7	V			15.34	11.7	33.6	0.316	0.364
330	30/12/1997	31.3	35.2	V			12.51	12.4	22.7	0.281	0.367
580	2/2/2003	32.6	51.7	V	V		14.66	11.9	29.8	0.306	0.368
329	30/12/1997	31.3	34.6	V			12.53	12.4	22.8	0.283	0.369
325	3/9/1997	31.8	34.7	V			18.3	11.2	44.6	0.368	0.369
241	28/4/1987	30.6	-104	V	V		12.46	12.5	21.5	0.279	0.37
674	18/7/2004	35.7	51.3	V	V		14.31	12.1	27.7	0.308	0.377
180	13/7/1980	41.4	-70.7	V			20.89	10.7	60.1	0.452	0.377
576	3/1/2003	32.4	-111	V			15.63	11.7	34.9	0.335	0.378
563	5/11/2002	32.4	-111	V	V	V	16.56	11.5	41.1	0.365	0.382
539	11/6/2002	32.4	-111	V	V		13.96	12.2	27.7	0.313	0.383
479	19/8/2001	33.9	-118	V	V		15.07	11.9	34.4	0.342	0.386
440	25/1/2001	40.8	-74		V		15.35	12	31.5	0.333	0.389
158	9/1/1978	30	-90.2	V	V		12.56	12.6	23.9	0.313	0.395
159	9/1/1978	30	-90.2	V	V		12.56	12.6	23.9	0.313	0.395
441	25/1/2001	40.4	-74.5	V			15.37	12.1	31.6	0.343	0.399
564	5/11/2002	32	-117	V			16.76	11.6	42.1	0.389	0.402
562	5/11/2002	26	-80.3		V		15.43	12	35.7	0.364	0.403
112	25/5/1933	55.6	33.9	V			15.67	12.1	32.8	0.353	0.403
464	22/6/2001	43.9	18.4	V			17.7	11.4	46.7	0.412	0.404
636	24/12/2003	32.7	51.7	V			17.05	11.6	43.5	0.399	0.405
641	22/1/2004	41.8	-123	V	V		16.85	11.7	41.5	0.391	0.406
671	18/6/2004	36.8	-81.8		V		13.88	12.6	25.8	0.331	0.406
243	28/4/1987	40.7	-112	V			13.08	12.7	23.6	0.324	0.408
399	6/2/2000	-4	39.7	V			13.07	12.7	23.6	0.327	0.41
459	24/4/2001	32.5	3.7	V	V	V	14.47	12.4	29.7	0.349	0.411
140	21/12/1976	29.9	-81.3	V			12.8	12.7	24.4	0.33	0.411
395	7/1/2000	10	-61.5	V			12.83	12.8	22.2	0.324	0.412
711	15/10/2004	30.2	57.1	V	V	V	19.12	11.2	53.3	0.452	0.412
530	13/5/2002	32.6	51.7	V	V		14.02	12.5	27.2	0.34	0.412
215	21/1/1985	19	-155	V			13.88	12.6	26.7	0.339	0.412
307	25/9/1995	-34	18.4	V			12.81	12.7	23.7	0.329	0.412
675	18/7/2004	32.7	51.7	V	V		14.25	12.5	27.5	0.343	0.414

568	5/12/2002	32.6	51.7	V	V		17.01	11.7	42.4	0.403	0.414
Obs.	Date	Lat	Long	VISIBILITY			ARCL	ARCV	Width	Model	
No.		deg	deg	N	B	T	deg	deg	Arc sec	q-value	s-value
379	10/10/1999	24.5	46.5		V		13.99	12.6	26.7	0.346	0.419
717	13/11/2004	13.7	10.7	V			15.52	12.1	36	0.385	0.423
446	24/2/2001	36	50.8	V			14.8	12.5	29.6	0.362	0.424
493	17/10/2001	10.3	9.8	V			13.09	12.8	25.3	0.347	0.425
205	27/8/1984	15.6	35.6				13.26	12.7	26.7	0.353	0.426
278	25/4/1990	41.6	-73.7	V		V	12.82	12.8	25	0.348	0.426
100	4/8/1921	-34	18.5	V			12.83	12.8	25	0.35	0.429
161	9/1/1978	29.7	-98.1	V			12.85	12.9	25	0.353	0.431
526	13/4/2002	26	-80.3	V			13.7	12.8	25.4	0.355	0.432
531	13/5/2002	33.3	44.4	V	V		14.27	12.7	28.2	0.365	0.433
380	10/10/1999	34	-6.8	V			15.55	12.4	32.9	0.383	0.433
465	22/6/2001	38.2	46	V	V		16.44	12.1	40.3	0.416	0.436
532	13/5/2002	29.6	52.5	V			13.94	12.8	26.9	0.364	0.436
244	28/4/1987	37	-122	V			13.13	13	23.8	0.358	0.44
448	24/2/2001	-34	18.4	V			16.14	12.4	35.2	0.399	0.44
554	7/10/2002	32.5	51.7	V	V		16.56	12.1	41.5	0.428	0.443
92	1/4/1919	53.9	-1.6	V			13.51	12.9	27.7	0.38	0.449
696	15/9/2004	-34	18.4	V			13.74	12.9	26.8	0.377	0.449
344	18/1/1999	6.5	3.4				13.3	13	25	0.372	0.45
116	14/5/1934	55.6	33.9	V			15.22	12.7	31.6	0.399	0.454
354	18/3/1999	34	-6.8	V			14.3	12.8	30.5	0.396	0.455
533	13/5/2002	29.4	48	V	V		14.09	12.9	27.4	0.385	0.456
129	6/4/1970	48	-122	V			13.29	13.1	25.7	0.381	0.457
1	1/7/1859	38	23.7	V			16.42	12.3	40.7	0.44	0.458
466	22/6/2001	35.7	51.3	V	V		16.16	12.4	38.9	0.438	0.464
733	11/1/2005	32.6	51.6	V	V		16.38	12.3	40.6	0.446	0.465
652	21/3/2004	33.9	-118	V			13.94	13.1	27	0.395	0.467
6	12/3/1861	38	23.7	V			13.37	13.3	24	0.386	0.468
124	5/3/1954	44.5	-88	V			13.15	13.2	26.1	0.394	0.469
454	24/2/2001	32.6	51.7	V	V		14.79	13	29.6	0.407	0.469
197	2/4/1984	15.6	35.6				13.42	13.3	24.4	0.391	0.471
534	13/5/2002	26.2	50.5	V	V		13.96	13.1	26.9	0.399	0.472
133	25/4/1971	39.5	-88.2	V	V		13.22	13.2	26.1	0.4	0.475
78	12/3/1899	52.5	13.3	V			13.32	13.2	26.2	0.401	0.476
527	13/4/2002	32.4	-111	V	V		14.49	13.1	28.4	0.411	0.478
536	13/5/2002	31.9	35.8	V		V	14.53	13.1	29.2	0.416	0.48
447	24/2/2001	5.3	103	V	V	V	13.4	13.4	24.3	0.399	0.48
535	13/5/2002	25.3	49.7	V			13.97	13.2	27	0.409	0.481
423	28/10/2000	32.6	51.7	V			15.81	12.9	34.8	0.446	0.489
13	1/1/1862	37.9	22.9	V			14.57	13.1	31.2	0.434	0.491
718	13/11/2004	10.3	9.8	V			15.6	12.8	36.4	0.456	0.492
366	14/6/1999	6.5	3.4	V			14.08	13.3	29.8	0.439	0.501
588	2/4/2003	33.8	-118	V			14.54	13.4	28.3	0.436	0.504
449	24/2/2001	29.6	52.5	V			14.79	13.3	29.5	0.442	0.505
136	5/3/1973	40	-85	V	V		13.56	13.5	26.6	0.436	0.509

141	21/12/1976	37.6	-123	V			14.14	13.5	29.6	0.457	0.519
Obs.	Date	Lat	Long	VISIBILITY			ARCL	ARCV	Width	Model	
No.		deg	deg	N	B	T	deg	deg	Arc sec	q-value	s-value
117	14/5/1934	50	36.2	V			14.91	13.4	30.4	0.462	0.522
577	3/1/2003	10.4	-61.5	V			14.27	13.5	29.1	0.458	0.523
126	5/4/1962	-26	-28.2	V	V		15.1	13.2	34.2	0.477	0.523
548	9/8/2002	10.3	9.8	V			13.92	13.7	28.9	0.478	0.543

In this table observation numbers given in first column are those assigned by Odeh (Odeh, 2004). The second column contains the date of observation. Next two columns give latitude (negative for south) and longitudes (negative for west), respectively. The three columns under the heading VISIBILITY contains V if crescent was visible. First of these is for naked eye visibility and the next two are for visibility with the help of a binocular or telescope. These are followed by columns containing ARCL (arc of light that shows separation of crescent from the sun), ARCV and crescent width. Both ARCL and ARCV are in degrees and crescent width is in arc seconds. The last two columns are for *q*-values define in (2) above and *s*-value as defined by (5).

Out of these cases only 11 cases of crescent sighting are not consistent with our model. Out of these 11 cases 9 are also not consistent with Yallop's criterion. The observation number 416 that is not consistent with other models is consistent with our model. The reason is that in this case ARCV is reasonably high (9.41 degrees) crescent width is small (around 14 arc seconds) but the Moon is very close to perigee so closest to the Earth. These factors make this very thin crescent significantly brighter. On the basis of a detailed analysis calculated s-values for the observational data we note that:

i) The crescent was never seen for *s*-value less than -0.16. The exception is the observation no. 389 (January 7, 2000 from 34^0S and 18^0.4E) that we reject as an authentic observation since it is not

ii) allowed by any of the ancient, medieval and modern criteria. *Thus whenever s-value < -0.16 the crescent can not be seen with or without optical aid.*

iii) For -0.16 < *s*-value < -0.06 there are 26 (45%) claims of crescent visibility with optical aid out of 58 reported and considered in this study. Only two (3%) such claims are without optical aid (455 of March 25, 2001 from 34^0S and 18^0.4E and 274 of February 25, 1990 from 35^0.6N and 83^0.5W). Observation No. 455 is not consistent with any model whereas observation no. 274 is allowed by only magnitude contrast model (Qureshi, 2007) and Schaeffer's model (Schaeffer, 1988b). Both these later models take in to consideration the site elevation and weather conditions (atmospheric temperature and the relative humidity) directly and not indirectly as is done in the model developed in this work and the one due to Yallop. Thus we deduce that *for -0.16 < s-value < -0.06 the crescent can be seen only with optical aid.*

iv) In 49 cases with -0.06 < *s*-value < 0.05 there are 13 naked eye sightings (25.5%) and 21 sightings are with optical aid (41%). We conclude that there are strong chances of sighting crescent with a binocular or a telescope and very slim chances for unaided sighting. Unaided sighting is not impossible. In most of the naked eye visibility cases crescent was seen after it was seen with optical aid. All these visibility claims were consistent with other models. Thus we deduce that for this range of s-values crescent visibility may require optical aid to first locate it and then visibility without optical aid is possible.

v) For 0.05 < *s*-value < 0.15, there are 35 sightings with optical aid (70%) and 14 without optical aid (28%). Thus the crescent may be easily seen with optical aid for *this range of s-value and can be seen without*

vi) *optical aid under very good condition* (weather conditions and height above sea level).

vii) For *s*-value > 0.15, out of next 213 observations the crescent was seen without optical aid 165 times (77.5%). Therefore we conclude that *for s-value > 0.15 the crescent can be easily seen.*

On the basis of this analysis of observational data in view of *s*-values the new criterion that we suggest is summarized in the **(Table 6)**. The figure 3 shows that straight line joining the turning points of the minimum visibility curves intersects the origin of the (*h, s*) coordinate system and has slope ((*h* + *s*)/*s* =) 9.3/5 or *h*/*s* = 4.3/5. This leads to a modified "best time" of crescent visibility as:

$$T_B = T_S + \frac{4.3}{9.3} LAG \qquad (6)$$

Table No. 6: New Visibility Criterion

Easily Visible (EV)	*s*-value > 0.15
Visible under perfect conditions (VUPC)	0.05 < *s*-value < 0.15
May require optical aid to find crescent (MROA)	-0.06 < *s*-value < 0.05
Require optical aid (ROA)	-0.16 < *s*-value < -0.06
Not visible with optical aid (I)	*s*-value < -0.16

Thus in our model the best time of crescent visibility is given by (6) that gives 1/(2.163)th part of LAG in comparison to 1/(2.25)th part of LAG in Yallop's criterion.

3. Discussion

The criterion summarized in the table - 5 has been developed on the basis of more recent and more accurate brightness models for twilight sky when compared to the criterion of Yallop (Yallop, 1998). This criterion also leads to the development of constant *s*-value pseudo parabolic curves that divide the globe into four regions in terms of crescent visibility conditions. These curves correspond to s = 0.15 (the curve A), s = 0.05 (the curve B), s = -0.6 (the curve C) and s = -0.16 (the curve D). Computer software like Mooncal (Manzur, 2001) can be used to show these regions. Such a division of the globe is only a guide for the new crescent searchers. It also serves to verify visibility claims of the observers that are not technically trained. Such verification is required in societies (particularly Muslims') where the verified claims are used to make decisions to begin a lunar month.

This class of criteria, including the Yallop's criterion, is still based on some averages (values of h are averages taken from a number of observations of crescents of same widths in this work) therefore these criteria are still not definitive. With a variety of site elevations, different weather conditions, seasons and earth-moon distance deviation may occur. For more definitive criteria, models that estimate and use accurate brightness of twilight sky and take into account accurate weather conditions and site elevation directly, ought to be better. However, such models require more accurate weather prediction systems and involve much more complex computations.

Reference

Bemporad, A. (1904) "On the Theory of Extinction", *Mitt. Sternw. Königstuhl-Heidelberg*: (**4**): Pp.69.

Blackwell, H. R. (1946) "Constant Thresholds of Human Eye", *J. Optical Soc. Amer.*, **36 (11)**: Pp. 624.

Bogan, L. (2004) "Astronomical Visual Minimum Magnitude Calculation", http://www.go.ednet.ns.ca/~larry/astro/vislimit.html.

Bretagnon, P. and G. Francou, (1988) "Planetary Theories in rectangular and spherical variables. VSOP87 solutions", *Astron. Astrophys.*, **(202)**: 309-315.

Bruin, F. (1977) "The First Visibility of Lunar Crescent", *Vistas in Astronomy*, **(21)**: Pp.331.

Chapront-Touzé, M. and J. Chapront, (1991) "Lunar Tables and Programs from 4000 B.C. to A.D. 8000", Willmann-Bell, Richmond, Virginia.

Manzur, A. (2001) "Moon Calculator: Version6.0", www.ummah.org.uk/mooncalc.html.

Meeus, J. (1998) "*Astronomical Algorithms*", 2nd Ed., Willmann-Bell Inc., Richmond, Virginia.

Odeh, M. S. (2004) "New Criterion for Lunar Crescent Visibility", *Experimental Astronomy*, **(18)**: 39-64.

Qureshi, M. S. (2007) "On the Comparative Study of Mathematical Models for Earliest Visibility of the Crescent Moon and their Modification", Ph.D. Thesis, University of Karachi.

Schaefer, B. E. (1988) "An Algorithm for Predicting the Visibility of Lunar Crescent", Proceedings of the Lunar Calendar Conference, Ed. Ahmed, I., Intern. Inst. Islamic Thought, Hendon VA. 11-12.

Yallop, B. D. (1998) "A Method of Predicting the First Sighting of the New Crescent Moon", Technical Note No. 69, HM Nautical Almanac Office, Royal Greenwich Observatory, Cambridge, UK. NAO.